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Performance of Signal-to-Noise Ratio Estimator with Adaptive Modulation

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Abstract — **Adaptive modulation techniques in wireless communications are paramount technologies designed to thrive in fading and noisy environments. Methods as such require an accurate estimate of the channel condition at the receivers' end in order for the transmitter to adapt appropriately. With that in mind, a communication link which changes the degree of modulation scheme according to the estimated signal-to-noise ratio (SNR) values is proposed. Hence, the objective of the system is to achieve acceptable quality in a changing communication link. In this paper, spectral efficiency and average bit error rate of the overall system is measured. Monte Carlo simulations of different signals and channel conditions corroborate our analysis and discussion.**

Keywords – Adaptive modulation and coding; SNR estimation; BER; Communication systems

I. INTRODUCTION

The necessity for reliable and high speed wireless communications is growing as the number of wireless devices are also growing synonymously. The ability to stay connected wherever we go has become essential if not standard. In that sense, given the variable channel conditions in reality, there will be a need maximize the spectral efficiency (SE) while minimizing the bit error ratio (BER). There are various methods to maintain efficient spectrum usage. An example is adaptive channel assignment techniques using intelligent algorithms [1]. However, this paper focuses on maximizing a singular channel. Adaptive modulation and coding (AMC) allows us to do just that by adjusting data rate, transmit power, instantaneous BER, and channel code rate in order to overcome noise and fading effects [2]. Also another challenging problem is to find the balance between which parameters to adapt for best performance. These adaptive techniques are advantageous for next generation wireless systems.

Channel state information (CSI), crucial to adaptive modulation, is derived from the receiver by estimating the instantaneous SNR. The transmitter then can use the CSI to decide what modulation parameters to employ, unlike fixed traditional systems designed to handle worst case conditions [3]. Most literature does not elaborate how the transmission scheme will utilize the feedback link. Most assumes a special case of non-ideal or ideal channel estimation information plus a feedback path with or without delay. The effects of non-ideal estimation and feedback delay are analyzed in [4, 5, 6].

SNR estimation provides valuable assistance in modern wireless communication systems. It can complement with adaptive modulation and coding schemes to enhance the channel quality in varying conditions. Other applications of SNR information include: information war, threat analysis, and electronic surveillance system.

Generally, SNR estimators are categorized to Data-Aided (DA) and Non-data Aided (NDA). Data-Aided estimators obtain knowledge of the transmitter sequence by receiving pilot symbols periodically. Whereas, NDA estimators do not require such knowledge. In another category, estimators are separated by I/Q and envelope based estimators, I/Q estimators processes in-phase and quadrature components of the received signal, thus synchronization required. In comparison, envelope-based estimators only focus on the received signal magnitude, and thus can be applied even if the carrier phase information is not available. The maximum likelihood (ML) SNR estimator provides good statistical performance but it comes with higher computational complexity compared to envelop-based estimators [5]. Reference [7] also provides insight to the various estimators under normalized simulation rules, and in summary, the authors summarized that the best estimator actually depends on the application in context. Besides, the implementation for an envelope-based estimator is much simpler. In this paper, an envelope moment based SNR estimator is combined with the adaptive modulation scheme.

The subsequent contents of the paper are arranged as follows. Section II provides a literature study. Section III introduces the system model and SNR estimation method. Section IV covers the adaptive modulation scheme with SNR estimator and in section V the results of the adaptive modulation scheme are displayed and discussed. Finally, section VI concludes the paper.

II. ADAPTIVE MODULATION WITH PERFECT CHANNEL STATE INFORMATION

The vast literature of adaptive communication systems span from as far back in the late 1960's [8]. Thereafter, possibly, due to lack of technical depth in estimation methods and practical constraints, the interest regarding said field started to gain momentum after approximately two

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decades. Goldsmith and Varaiya, in their work [9], showed that Shannon capacity of the communication link is achieved by adapting the rate and power of transmitter. This approach was extended to [3], where a practical variable-rate variable power system is derived and analysed. The adaptive system's spectral efficiency is fulfilled while satisfying BER and power boundaries. It is also shown that the technique has up to 10 dB gain over constant rate variable power modulation whereas an even bigger 20 dB gain over non-adaptive modulation.

Following the gain in discoveries of the advantages of adaptive modulation schemes, a proposal to implement AMC in cellular wireless standards was given in [10], specifically in the CDMA2000 standard. Furthermore, AMC is also imparted in WCDMA high speed downlink packet access (HSDPA), IEEE 802.11x family standards and also WiMax standards for wireless internet access.

III. MOMENTS-BASED SNR ESTIMATION

This section introduces a novel envelope-based NDA estimator from [11]. This estimator applies to constant modulus (CM) modulations and non-constant modulus cases. In this paper, only CM modulation schemes are considered. [9] showed that by adapting the rate and power of the transmitter, the transmission link is able to achieve close to the Shannon capacity.

A. System Model

Assume a discrete channel model, so that the sampled symbol at the receiver's matched filter output given by

$$r_k = \sqrt{g_k}\, x_k + x_k\ ,\ k = 1, \ldots, K \tag{1}$$

Where x_k represents the transmitted complex symbols, $\sqrt{g_k}$ is stationary and ergodic channel gain, and n_k are independent and identically distributed Gaussian noise with unknown variance $2\sigma^2$. With samples $y_0, y_1, \ldots, y_{K-1}$ the system is predicted to estimate the SNR value. This paper's envelop-based SNR estimator relies on $|y_k|$.

Adaptive modulation's system model is illustrated in Fig. 1.

B. Second- and Fourth-Order Moments (M_2M_4) NDA Estimator

The symbols are drawn from a finite constellation which is known to the receiver and has I different amplitudes, having i^{th} amplitude A_i and probability P_i ($I = 1, \ldots, I$). The constellation p^{th} moment is denoted by

$$M_p = E\{|\, x_k\, |^p\} = \sum_{i=1}^{I} P_i A_i^p \tag{2}$$

It is mentioned in [12], sample moment of the envelope are measured by their respective time averages as the following function with K_{sym} as the total number of received symbols:

$$M_p \approx \frac{1}{K_{sym}} \sum_{k=1}^{K_{sym}} |r_k|^p \tag{3}$$

From which an estimate if the SNR, $\rho = S/N$. The equation of true moment of the envelope is given by [11]

$$M_p \approx (2\sigma^2)^{\frac{p}{2}} \sum_{i=1}^{I} P_i \Gamma\left(\frac{p}{2}+1\right) e^{-\rho A_i^2}\, {}_1F_1\left(\frac{p}{2}+1; 1; \rho A_i^2\right) \tag{4}$$

Where ${}_1F_1(\bullet\ ;\ \bullet\ ;\ \bullet\ )$ is the confluent hypergeometric function, and $\Gamma(\bullet)$ is the gamma function. From (4), it can be observed that ρ and σ are related to the moment. Since, the moment-based estimator uses at least two different moments. Suppose $k \neq l$, let functions of ρ be:

$$f_{k,l}(\rho) := \frac{M_k^l(\sigma^2, \rho)}{M_l^k(\sigma^2, \rho)} \tag{5}$$

Note that (5) depends on A_i, ρ, and p_i but not on standard deviation, σ. Then the moments-based SNR, ρ estimator is expressed as

$$\hat{\rho}_{k,l}(\rho) := f_{k,l}^{-1} \frac{M_k^l}{M_l^k} \tag{6}$$

In [11], to overcome the tractability of the inverse function f, a look-up table was proposed to easily compute the estimation formula. However, in this paper, values of

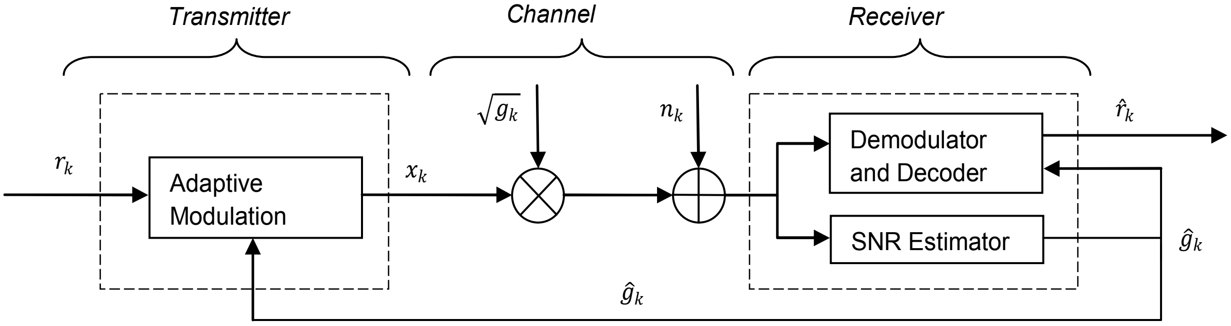

Figure 1. System model with feedback.

k = 2 and *l = 4* are used as we are only dealing with second and fourth moments. Thus, its inverse is possible and resulting in closed form solution given by:

$$\hat{\rho}_{2,4} := \frac{1 - 2\frac{M_2^2}{M_4} - \sqrt{(2-a)(\frac{2M_2^4}{M_4^2} - \frac{M_2^2}{M_4})}}{a\frac{M_2^2}{M_4} - 1} \tag{7}$$

Where, $a = \sum_{i=1}^{I} p_i A_i^4$. For PSK constellations, (7) is equivalent to the M_2M_4 SNR estimator for complex signals in [7]. Whereby,

$$\hat{\rho}_{2,4} := \frac{\sqrt{2M_2^2 - M_4}}{M_2 - \sqrt{2M_2^2 - M_4}} \tag{8}$$

IV. ADAPTIVE MODULATION WITH SNR ESTIMATION

A. Adaptive M-Psk Modulation

In this proposed model shown in Fig. 1, there is a delay- and error-free feedback path to the transmitter. This feedback is responsible for returning CSI of SNR estimates. Let γ_k denote the instantaneous SNR received at time *k*. The received signal is also assumed to have ideal coherent phase. Since g_k is stationary, then it is also independent on time *k*, we denote this distribution *p(γ)*.

The receiver adapts the modulation constellation according to SNR received from the feedback. Given a finite set of constellations available, $\{\gamma_i\}_{i=0}^{N-1}$ defines the range of γ where the constellations are associated. One constellation is assigned to an instantaneous SNR region of $[\gamma_i, \gamma_{i+1})$ ($0 \le i \le N-1$), but when an SNR value drops below γ_0, the communication link stops transmitting. γ_0 is called the cut-off SNR. This means that if the channel reaches an extensive degrade in quality, the channel should not be used. In this paper, -6 dB is the cut-off value as it is decided by the performance if the SNR estimator. This paper considers only $0 \le i \le 2$, where $M_i = \{4, 8, 16\}$.

B. Channel Capacity

Firstly, the channel capacity of a wireless transmission over AWGN channel is described by the equivocation theory,

$$C = \max_{f_X(x)}\{H(R)\} - H(N) \tag{9}$$

Where *H(N)* is the entropy of additive white Gaussian noise and entropy of the output *H(R)* is:

$$H(R) = -\int_{-\infty}^{\infty} f_R(r)\log_2 f_R(r)dy \tag{10}$$

The entropy if the white Gaussian noise if given by:

$$H(N) = \frac{1}{2}\log_2(\pi e N_0) \tag{11}$$

Depending in the channel, we describe our channel capacity in the case of discrete time AWGN with finite modulation constellations by combining Eqs. (9, 10, 11):

$$C = -\int_{-\infty}^{\infty} f_R(r)\log_2 f_R(r)dy - \frac{1}{2}\log_2(\pi e N_0) \tag{12}$$

The probability of the output function is given by:

$$f_R(r) = \frac{1}{|M|}\sum_x \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-\left(\sqrt{g}x+n\right)^2}{2\sigma^2}} \tag{13}$$

During transmission, besides factors like transmit power and coding, for a finite alphabet input *M* there is a limit to how many bits can be transferred at given time. It can be concluded immediately that:

$$C = \log_2 M \text{ for } \frac{S}{N} \to \infty \tag{14}$$

C. Assumptions and Performance Measure

In this paper, Spectral efficiency (SE) and average BER is evaluated. SE, denoted R/B, is equals average data rate over unit bandwidth. At point in time, $k(\gamma) = \log_2[M]$ bits/symbol is sent across the channel where M-ary PSK is used, this also equals the spectral efficiency for a fixed *M*. Typically, Spectral efficiency is influenced by BER and SNR. The Shannon-Hartley equation, (15), describes just that and BER is a factor because it relies on the theories of mutual information and entropy, which are not discussed in this general formula. Where *C* is the channel capacity in unit bps.

$$\frac{C}{B} = \log_2(1 + \frac{S}{N}) \tag{15}$$

For a discrete number of constellations the spectral efficiency is given by:

$$\frac{R}{B} = \sum_{i=0}^{N-1} k_i \int_{\gamma_i}^{\gamma_i+1} p(\gamma)d\gamma \text{ bps/Hz} \tag{16}$$

For BER it is assumed as:

$$\overline{\text{BER}} = \frac{E[\text{number of error bits per transmission}]}{E[\text{number of bits per transmission}]} \tag{17}$$

The Symbol Error Rate (SER) expression for M-psk in AWGN channel with grey mapping is described in [12]:

$$\text{SER}_{\text{M-psk}}(\gamma) = 2Q(\sqrt{2\gamma}\sin(\frac{\pi}{M})) \tag{18}$$

Due to grey mapping, two adjacent symbols differ only in a single bit. Therefore, the most probable case of selecting the neighbouring symbol in the event of noise distortion k-bit symbol can only have 1-bit error. So the approximation of $\mathrm{BER}_{\text{M-Psk}}$ is:

$$\mathrm{BER}_{\mathrm{M-psk}} \approx \frac{1}{k}\, erfc(\sqrt{\gamma}\sin(\frac{\pi}{M})) \tag{19}$$

However, this expression is not easy to work upon with its arguments. In that case, we resort to approximation of another simpler BER expression using curve fitting while keeping it in tight bounds. In [2], an approximation of BER for M-Psk constellations that is valid for $k(\gamma) \geq 2$ within 1.5dB of error for $\mathrm{BER} \leq 10^{-3}$ is adapted as:

$$\mathrm{BER}_{\mathrm{M-psk}} \leq 0.05 e^{\left(\frac{-6\gamma}{2^{2M}-1}\right)} \tag{20}$$

Rearranging (20), we obtain an expression to maintain the BER while producing the possible maximum constellation size:

$$\mathrm{M}(\gamma) = \sqrt{\frac{-6\gamma}{\ln(20\mathrm{BER})} + 1} \tag{21}$$

The spectral efficiency that is bounded by (14) is hence obtained by substituting (21) into the expression $R/B = \log_2 M$, Resulting in the expression:

$$\frac{R}{B} = \log_2\left(\sqrt{\frac{-6\gamma}{\ln(20\mathrm{BER})} + 1}\right) \tag{22}$$

V. RESULTS AND ANALYSIS

The M_2M_4 moments NDA SNR estimator performs very well from –5 dB onwards for BPSK, QPSK and 8-PSK. The estimator's performance is mainly acceptable. However, estimation values below –5 dB are not desirable. In Fig. 2, at –10 dB shows approximately error of 4 dB. From that point onwards, its performance gradually improves with better SNR values. It is also pointed out that the M_2M_4 does not discriminate between all 3 types of M-psk modulation. The performance is about the same for all. In real world implementation, the error of the estimator in low SNR will affect adaptive M-psk modulation which in reality the channel conditions are bad, but the estimator gives a better SNR feedback value to the transmitter. In the end, that can contribute to a higher BER.

Besides having the best performance compared to other types of estimators as shown in [7], the complexity of the M_2M_4 estimator is a factor to consider. Advantageously, simulation implementations for this estimator are not complex, therefore it computes rapidly.

Comparing the BER performance between the fixed MPSK modulation methods with adaptive modulation in Fig. 3, we see that in low SNR values (-5 dB until 4 dB), adaptive modulation has a better BER performance compared to fixed 16 PSK modulation scheme. For adaptive modulation, there is advantage in having low error rate at low SNR conditions and also higher bit rate performance at favorable channel conditions. This ensures that reliable communication is maximized no matter in good or bad channel conditions.

In Fig. 4, the SE at (22) is evaluated with BER constraints of 10^{-3} and 10^{-6} in comparison with Shannon capacity. Both BER limited curves of adaptive M-psk modulation are still below the Shannon limit curve which means it is still inside the practical region. As the curves strive to reach closer to the Shannon limit, it requires effective coding techniques and spectrum efficient efforts.

The channel capacity between adaptive modulation and M-psk modulation methods are compared. It can be observed in Fig. 5 that adaptive modulation reaches its maximum capacity (14) as SNR increases. Besides, its performance is also similar to that of 16 PSK because adaptive M-psk modulation used in this paper with finite alphabet only goes to a maximum cardinality of 16 at

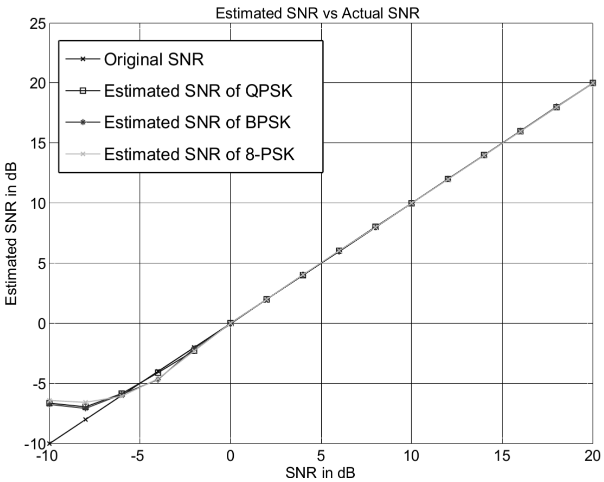

Figure 2. Comparison of estimated SNR with actual SNR.

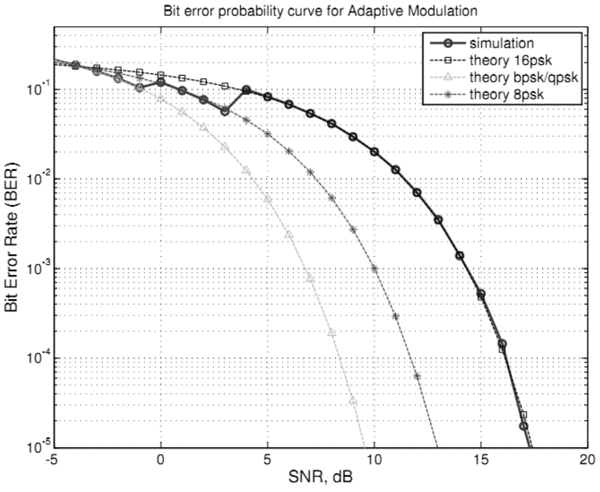

Figure 3. BER performance of adaptive M-psk modulation.

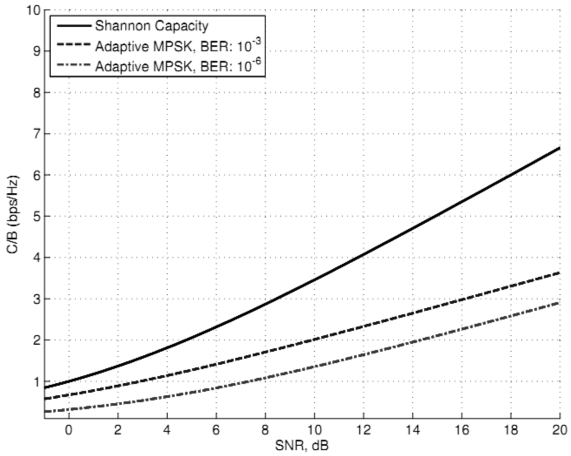

Figure 4. Spectral efficiency of adaptive MPSK modulation in AWGN.

favorable conditions. However, do note that the BER performance of adaptive modulation is better at lower SNR values due to its adaptive nature. So, at higher SNR situations it performs close to a fixed 16 PSK modulation scheme.

VI. CONCLUSION

In this paper, second- and fourth-order moments based SNR estimator proves to be reliable, simple and accurate. Envelope-based SNR estimators are less complex yet produce excellent performance for M-psk constellations. This qualifies the estimator to complement the adaptive system described. We have shown that for a certain range of low SNR environment, the BER performance is much better in adaptive modulation compared to fixed modulation schemes. We also compared the spectral efficiency of said system with the theoretical bound.

However, there are many constraints to take account before considering it as a practical implementation. This paper provides analytical approach towards adaptive modulation's capabilities in the context of measuring overall performance with assumptions.

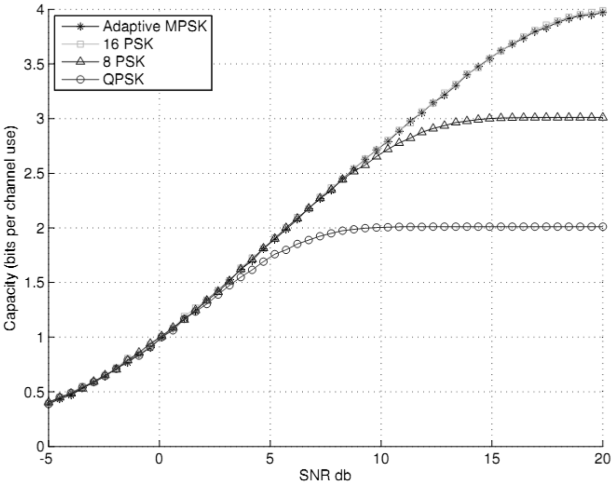

Figure 5. Channel capacity comparison with fixed bandwidth.

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REFERENCES

[1] Y. S. Chia, Z. W. Siew, A. Kiring, S. S. Yang and K. T. K. Teo, "Adaptive hybrid channel assignment in wireless mobile network via genetic algorithm," *11th International Comference on hybrid Intelligent Systems,* Dec. 2011, pp. 511-516.

[2] S. T. Chung and A. J. Goldsmith, "Degrees of freedom in adaptive modulation: A unified view". *IEEE Transactions on Communications,* vol. 49, no. 9, Sept 2001, pp. 1561-1571, doi: 0.1109/VETECS.2001.944588.

[3] A. J. Goldsmith and S. Chua, "Variable-rate variable-power MQAM for fading channels," *IEEE Transactions on Communications,* vol. 45, No. 10, Oct 1997, pp. 1218-1230, doi: 10.1109/26.634685.

[4] A. J. Goldsmith and L. Greenstien, "Effect of average power estimation error on adaptive MQAM modulation," *Proceedings of IEEE ICC '97,* Montreal, Que., Canada, 1997, pp. 1105-1109, doi: 10.1109/ICC.1997.610059.

[5] M. Alvarez-Diaz, R. Lopez-Valcarce and C. Mosquera, "SNR estimation for multilevel constellations using higher-order moments," *IEEE Transactions on Signal Processing,* vol. 58, no. 3, March 2010, pp. 1515-1526, doi: 10.1109/TSP.2009.2036069.

[6] M. Mohammad and R. M. Buehrer, "On the impact of SNR estimation error on adaptive modulation," *IEEE Communication Letters*, vol. 9, no. 6, Jun 2005, pp. 490-492, doi: 10.1109/LCOMM.2005.1437347.

[7] D. R. Pauluzzi and N. C. beaulieu, "A comparison of SNR estimation techniques for the AWGN channel," *IEEE Transactions on Communications,* vol. 48, no. 10, Oct 2000, pp. 1681-1691, doi: 10.1109/26.871393.

[8] J. F. Hayes, "Adaptive feedback communications," *IEEE Transactions on Communication Technology.,* vol. COM-16, pp 29-34, Feb. 1968.

[9] A. J. Goldsmith and P. Varaiya, "Capacity of fading channels with channel side information," *IEEE Transactions on Information Theory.,* vol. 43, no. 6, Nov 1997, pp. 1986-1992.

[10] P. Bender, P. Black, M. Grob, R. Padovani, N. Sindhushyana, S. Viterbi, "SDMA/HDR: a bandwidth efficient high speed wireless data service for nomadic users," *IEEE Communications Magazine,* vol. 38, no. 7, Jul 2000, pp 70-77.

[11] P. Gao and C. Tepedelenlioglu, "SNR estimation for non-constant modulus constellations," *IEEE Wireless Communications and Networking Conference (WCNC),* vol. 1, March 2004, pp. 24-29, doi: 10.1109/WCNC.2004.1311512.

[12] J. G. Proakis, *Digital Communications,* 2nd ed., McGraw-Hill, New York, 1989.