# Graph-based Image Segmentation using K-Means Clustering and Normalised Cuts

Mei Yeen Choong, Wei Leong Khong, Wei Yeang Kow, Lorita Angeline, Kenneth Tze Kin Teo Modelling, Simulation & Computing Laboratory, Minerals & Materials Research Unit School of Engineering and Information Technology Universiti Malaysia Sabah Kota Kinabalu, Malaysia msclab@ums.edu.my, ktkteo@ieee.org

Abstract—Image segmentation with low computational burden has been highly regarded as important goal for researchers. Various image segmentation methods are widely discussed and more noble segmentation methods are expected to be developed when there is rapid demand from the emerging machine vision field. One of the popular image segmentation methods is by using normalised cuts algorithm. It is unfavourable for a high resolution image to have its resolution reduced as high detail information is not fully made used when critical objects with weak edges is coarsened undesirably after its resolution reduced. Thus, a graph-based image segmentation method done in multistage manner is proposed here. In this paper, an experimental study based on the method is conducted. This study shows an alternative approach on the segmentation method using k-means clustering and normalised cuts in multistage manner.

Keywords-image segmentation; normalised cuts; k-means clustering

# I. INTRODUCTION

Image segmentation has been recognised as fundamental task in various image and video processing applications. Often, the purpose of image segmentation is to carry out decomposition of an image into meaningful segments such that these segments correlate with the real world and these are used for further important analysis. For example, in medical imaging field, segmentation plays an important role by locating certain anatomical part that a physician required to focus on which enables a proper treatment planning can be conducted in effective manner.

Exploitations on segmentation method approach lead to developing reliable segmentation techniques by many researchers. Two main segmentation approaches can be grouped for these segmentation techniques, local features based segmentation and global features based segmentation approach. Local features based approach such as Canny edge detector [1], uses local information for particular sets of pixels or surrounding pixels with central pixel in it [2]. However, edge detector has drawbacks. One of them is that it solely relies on local calculations of the particular set of pixels [3]. The boundary identifiers are not susceptible enough to detect critical edges of objects in an image. The misdetection of edges will result such that there is no guarantee for complete closed outline contours of a significant object in an image can be generated [4]. Unclosed contours will give undesirable segmentation results as distinctive features between different objects in an image are not clear.

Global features based image segmentation techniques are implemented by researchers to solve the limitation of edge detectors. An image should be viewed globally by simply indentifying the similarity and dissimilarity regions in an image. Image pixels that share common features such as brightness or colour are grouped as one region which indicating they belong to same object. One of the global features based image segmentation method is graph partitioning method by simply representing an image as a graph that can be partitioned into meaningful segments. With this method, distinctive features between regions are clear enough to enable the formation of closed contours of regions [5]. In this graph partitioning method, pixels are considered as nodes and there are connecting links between them. Each of the links for all paired nodes gives important similarity information between the nodes. Low similarities between two nodes are known as weak links are bound to be removed and these form disjointed segments. Normalised cuts algorithm is introduced here for the graph partitioning approach. K-means clustering is incorporated with this normalised cuts algorithm. To improve the computational efficiency as normalised cuts algorithm requires massive similarity measurement computation, the image segmentation is done in two stages [6].

The details of the multistage segmentation process in this paper are structured as followings. Section II described the normalised cuts algorithm and the flow of the segmentation by implementing it in multistage manner. Segmentation on synthetic images based on proposed approach is presented in section III. In section IV, the experimental results based on case-to-case natural image segmentation and discussions are provided. This paper is concluded in section V.

# II. MULTISTAGE IMAGE SEGMENTATION WITH NORMALISED CUTS

# A. Normalised Cuts

Normalised cuts algorithm is generally based on graph theoretic approach in which grouping similar pixels into homogeneous regions. A weighted graph G = (V, E) which represents an image contains the vertices (nodes), V and edges, *E*. The *E* is related to weights, *w*. The similarity measurement between node *i* and node *j* is denoted by *w*. The graph is then bi-partitioned into two distinct sub-graphs A and B with the condition that it minimizes the value of

$$cut(A,B) = \sum_{i \in A, j \in B} w(i,j), \qquad (1)$$

with constraint  $A \cup B = V$  and  $A \cap B = \emptyset$  [7, 8, 9]. Based on the dissimilarity measurement between two sub-graphs *A* and sub-graphs *B*, the weights of the pairs of nodes are to be removed is shown in (1). The bi-partitioning process is done recursively until a minimum cuts is met and *k* sub-graphs are formed. The criterion of the minimum cuts is such that the maximum possible cut across the sub-graphs is minimized. There is high similarity in the formed homogeneous sub-graphs and low similarity across different sub-graphs. There is weakness on this minimum cut criterion such that its cut algorithm favors in cutting isolated nodes when forming sub-graphs.

Another cut algorithm which is the normalised cuts (Ncut) is proposed to solve the problem [7, 8, 9]. The normalised cut criterion is derived as in

$$Ncut(A,B) = \frac{cut(A,B)}{assoc(A,V)} - \frac{cut(A,B)}{assoc(B,V)},$$
 (2)

whereby  $assoc(A, V) = \sum_{i \in A, j \in V} w(i, j)$ . The Ncut(A, B) is then transformed to

$$(D-W)y = \lambda Dy, \qquad (3)$$

to solve for the eigenvectors y and eigenvalues  $\lambda$ . D - W, a Laplacian matrix whereby W is a symmetrical matrix with W(i, j) = w(i, j). Each of the weight elements in matrix W is defined as

$$w(i,j) = e^{\frac{-\|\mathbf{F}(i) - \mathbf{F}(j)\|_{2}^{2}}{\sigma_{l}}} * \begin{cases} \frac{-\|\mathbf{X}(i) - \mathbf{X}(j)\|_{2}^{2}}{\sigma_{s}}, & \text{if } - \|\mathbf{X}(i) - \mathbf{X}(j)\|_{2}^{2} < r, \\ 0, & \text{otherwise} \end{cases}$$
(4)

where  $\mathbf{F}(i)$  is feature vector based on intensity value in colour of node *i* and  $\mathbf{X}(i)$  is the spatial location of the node. When a pair of node *i* and node *j* is more than r number of pixels apart the weight w(i, j) is considered 0. *D* is a diagonal matrix with  $d(i)=\sum_{j=1}^{j}w(i, j)$  on its diagonal [7, 8, 9].

Clustering is performed on the computed eigenvector. The computed eigenvector contains description value corresponds to each of the pixel in terms of spatial and pixel intensity measurement in the image. Eigenvector corresponds to the second smallest eigenvalue is chosen for a clustering algorithm to partition the eigenvector. After the clustering process on the eigenvector, segmentation result is finally obtained based on the clusters formed among the eigenvector. K-means clustering [10] is selected to perform the clustering. Mean-shift algorithm for the clustering is opted to be used for the clustering algorithm [11]. However, because of the mean-shift clustering performs much slower than the k-means although the value k are not needed as input parameter, mean-shift clustering is not selected for the clustering on the computed eigenvector.

### B. Two-Stage Segmentation with Normalised Cuts

Normalised cuts requires massive matrix of similarity measurement. Such large matrix is not ideally to be processed. To retain information in a high resolution image, the image can be divided into equal size of box-shaped image cells [12]. It is known that natural scene image has irregular pixels distribution pattern. A preliminary check is perform on every image cell to determine whether the particular cell requires normalised cuts algorithm since the frequency of image pixels is distributed unevenly.

The first stage of segmentation is run with the  $k_{l}$  number of clusters is provided. Although over-segmentation would likely to be occurred, it helps to reduce the tendency of object boundaries to be missed out. Fig. 1 shows an image is divided into designated number of cells and local segmentation on each of the cells. The segmentation on each of the cells is done independently. Segmented clusters from the image cells are then expected to be merged in second stage segmentation. In the second stage segmentation, simple representations of the clusters are computed by representing each cluster with one pixel node at centroid location of the cluster. Normalised cuts are also run on these computed nodes at the second stage segmentation. The value of maximum distance for a pair of pixels can be apart, r, and the standard deviation for the spatial location of the node,  $\sigma_s$  are different with the one in first stage because the computed nodes are scattered in sparse manner. The computed nodes are then taken for similarity measurement. Clusters are then merged together by grouping the nodes in a same region when they share common similarity based on color and spatial feature.

### III. SEGMENTATION ON SYNTHETIC IMAGES

To illustrate image segmentation based on the proposed approach, a simple  $40 \times 40$  synthetic image with checker pattern is used here to illustrate the various parameters settings that give significant influence to the segmentation performance.

## A. Effect of $\sigma_I$ in Segmentation

By implementing  $4 \times 4$  image cells segmentation, 4 clusters are well segmented out. Each cell contains 4 different colours are clustered separately for each colour. Fig. 2 shows its segmentation result with  $\sigma_I = 10$  in second stage segmentation. When  $\sigma_I$  is decreased until it is less than 1, no segmentation result can be produced. This is most likely due to its  $\sigma_I$  is too small such that the k-means clustering fails to converge. Increasing  $\sigma_I$  tends to produce



Figure 1.  $3 \times 3$  image cells and its segmentation on the image cells individually.

incorrect segmentation as it can be shown in Fig. 3. Notice that from  $\sigma_I = 1000$  to  $\sigma_I = 10000$ , there are no difference in the segmentation results. The optimal value  $\sigma I$  for correct segmentation is image content dependent when the colour in the image is altered with similar checker pattern and  $4 \times 4$  image cells. Fig. 4 shows the segmentation results with different colour combination.

# B. Effect of $\sigma_S$ in Segmentation

As normalised cuts algorithm used here is based on the colour similarity and spatial similarity, the effect  $\sigma_S$  on the segmentation has been experimentally tested. Similar initial settings mentioned in previous sub-section for the segmentation to be performed and  $\sigma_I = 10$  is used throughout the test. Various value  $\sigma_S$  have been used for the test. The segmentation results with various  $\sigma_S$  are shown in Fig. 5. It can be seen that the  $\sigma_S$  for correct covers a wide range even up to value of 5000. This most likely tells the increasing of  $\sigma_S$  until a stage such that it has no negative effect on the segmentation process.

# C. Effect of Number of Clusters Segmented Per Cell in Segmentation

In the first stage of local segmentation, since local segmentation take place on each of the cells, the number of



Figure 2. Segmentation with  $\sigma_I = 10$  in second stage segmentation.



Figure 3. Segmentation results with various  $\sigma_I$ .

clusters segmented per cell reflects how detail the segmentation of the image should be done.

Increasing the number of clusters per cell will reduce the number of pixels for each node to represent. It is idea that the number of clusters per cell to be increased such that the nodes will have less number of pixels to represent. Thus, this reduces the tendency of under segmentation. However, needless to say that the number of clusters for the image to be segmented out shall never be more than the number of clusters generated from the cells as the k-means clustering fails to converge whereby the number of centroids is equal to the number of segmented clusters of an image. Fig. 6 shows the node view in second stage segmentation, result and graph of eigenvector corresponding with the node by from two different numbers of clusters per cell implemented.

### IV. IMAGE SEGMENTATION ON NATURAL IMAGE

To illustrate image segmentation based on the proposed approach, natural images are shown in following figures.

# A. Segmentation on Image with Tiny Objects

Based on the Fig. 7, boundaries of the segmented cluster follow nicely with the edge of the planes in the image. The segmentation for the image whereby the image is divided



Figure 4. Segmentation results on different colour combination with similar checker pattern with various  $\sigma_I$ .



Figure 5. Segmentation results with various  $\sigma_s$  by applying  $\sigma_l = 10$ .

into numerous image cells takes about 295 seconds which is considered long.



Figure 6. Segmentation results with 2 different numbers of clusters per cell are implemented.

In the attempt to reduce computation time, generation of  $10 \times 10$  image cells is performed for the segmentation. Eventually, the computation takes about 35 seconds which is less than the previous one about 8.4 times of the computation time. The area of an image cell has become smaller although more cells are added for first stage segmentation. This shows

increasing number of cells helps to reduce the computation time. Small image cell is not appropriate for image with tiny objects in it. This leads to the segmentation on the large portion area of image is performed unnecessary. Fig. 8 shows the segmentation result when the number of clusters per cell is increased.

# B. Segmentation on Image with Large Object

Fig. 9 shows a plane being segmented out from the background. More detailed contents of the cell are segmented out, when the number of clusters per cell increased. The number of clusters per cell indicates the complexity of the image itself. High number of image clusters per cells indicates the image is made up of numerous



Figure 8.  $20 \times 20$  image cell and its segmentation result.

objects which allow more clusters needed to be segmented out. However, increasing the number of clusters per cell should be done cautiously to avoid segmentation to the extent where some of the edges in the image such as the object's shading and shadow are segmented out separately. Selecting an optimal number of clusters per cell should be based on the image content or application used to reduce the tendency of the image being segmented undesirably. The size of image cells can be inconsistent when there are certain objects in the image need more detailed segmentation on it.

# C. Segmentation on Image with Rich Boundaries

The common goal of image segmentation is conducted is to segment out objects in foreground from the background of an image. However, when the background of an image is rich with multiple colours and textures, it is hard to define which objects are to be segmented from background. This occurs when the object's edges seemingly appear to be part of the background. An image from the Berkeley Segmentation Dataset (BSDS500) [13] contains the steel roller coaster tracks reflects the issue and its result is shown in Fig. 10. Primitive image segmentation method may not perform well when the image contains unclear boundaries give more challenges to the researchers to improve. A highlevel knowledge may have to be incorporated in it to improve the method [14].



Figure 9.  $10 \times 10$  image cell segmentation, 3 clusters per cell, segmentation result and graph.



Figure 10. An image from BSDS500 and its segmentation result.

# V. CONCLUSIONS

This paper presents an experimental study on image segmentation with alternative approach for normalised cuts algorithm. To optimise the performance of normalised cuts algorithm is considered a NP-hard problem [4]. Thus, an alternative approach should be implemented to help reducing unnecessary image segmentation instead of performing segmentation on whole image alone. Segmentation on image part by part individually also helps to speed up the computation time of similarity measurement in normalised cuts algorithm.

This study leads to implement a method such it balances up the trade-off between efficiency and effectiveness of the normalised algorithm. Artificial intelligence technique such as neural network can be implemented as pre-processing technique [15]. This technique can adaptively fine tune parameters such as the image cell size and numbers of image cells according to the image content, foreground area that covers more than background area, the number of image cells can be such a way it is not many to reduce the tendency of waste in computation time in normalised cuts algorithm.

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#### REFERENCES

 J. Canny, "A Computational Approach to Edge Detection," IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 8, issue 6, 1986, pp. 679–698.

- [2] M. Sonka, V. Hlavac, and R. Boyle, "Image Processing, Analysis, and Machine Vision," Intenational Thomson Publishing, 1999.
- [3] M. Petrou and C. Petrou, Image Processing: The Fundamentals. John Wiley & Sons, 2010.
- [4] S.C. Zhu, and A. Yuille, "Region Competition: Unifying Snakes, Region Growing, and Bayes/MDL for Multiband Image Segmentation," IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 18, Sep. 1996, pp. 884-900, doi:10.1109/34.537343.
- [5] Z. Wu and R. Leahy, "An Optimal Graph Theoretic Approach to Data Clustering: Theory and Its Application to Image Segmentation," IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 15, Nov 1993, pp. 1101 -1113, doi: 10.1109/34.244673.
- [6] M.Y. Choong, C.F. Liau, J. Mountstephens, M.S. Arifianto, and K.T.K. Teo, "Multistage Image Clustering and Segmentation with Normalised Cuts," Proc. 2012 Third International Conference on Intelligent Systems, Modelling and Simulation (ISMS), February 2012, pp. 362-367, doi:10.1109/ISMS.2012.112.
- [7] J. Shi, and J. Malik, "Normalized Cuts and Image Segmentation," IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 22, Aug 2000, pp. 888 -905, doi: 10.1109/34.86868.
- [8] M. Martí nez P. Mittrapiyanuruk, and A. C. Kak, "On Combining Graph-Partitioning with Non-Parametric Clustering for Image Segmentation," Computer Vision and Image Understanding, vol. 95, July 2004, pp. 72-85, doi: 10.1016/j.cviu.2004.01.003.
- [9] L.G. Shapiro and G.C. Stockman, Computer Vision. Prentice Hall, 2001.
- [10] J.A. Hartigan, and M. A.Wong, "Algorithm AS 136: A K-Means Clustering Algorithm," Journal of the Royal Statistical Society, Series C (Applied Statistics), vol. 28, no. 1, 1979, pp. 100–108, doi:10.2307/2346830.
- [11] Y.S. Chia, W.Y. Kow, W.L. Khong, A. Kiring, and K.T.K. Teo, "Kernel-Based Object Tracking via Particle Filter and Mean Shift Algorithm," Proc. 2011 11th International Conference on Hybrid Intelligent Systems (HIS), December 2011, pp. 522-527, doi:10.1109/HIS.2011.6122159.
- [12] J. Malik, S. Belongie, T. Leung, and J. Shi, "Contour and Texture Analysis for Image Segmentation," International Journal of Computer Vision, vol. 43, June 2001, pp. 7-27, doi: 10.1023/A:1011174803800.
- [13] P. Arbelaez, M. Maire, C. Fowlkes, and J. Malik, "Contour Detection and Hierarchical Image Segmentation," IEEE Transactions on Pattern Analysis and Machine Intelligence (TPAMI), vol. 33, no. 5, May 2011, pp. 898-916, doi: 10.1109/TPAMI.2010.161.
- [14] S. Maji, N. Vishnoi, and J. Malik, "Biased Normalized Cuts," Proc. 2011 IEEE Conference on Computer Vision and Pattern Recognition (CVPR), June 2011, pp. 2057 -2064.
- [15] L. Angeline, W.L. Khong, F. Wong, I. Saad, and K.T.K. Teo, "Multiple Vehicles' License Plate Tracking and Recognition via Isotropic Dilation," Proc. 2011 Third International Conference on Computational Intelligence, Communication Systems and Networks (CICSyN), July 2011, pp. 54-59, doi:10.1109/CICSyN.2011.24.